

Geometry 2.3

Analyze statements in if-then form

Write the converse, inverse, and contrapositive of conditional statements

conditional statement

$$p \rightarrow q$$

biconditional

if and only if

iff

all related cond. T

hypothesis

conclusion

Quiz 2.1-2.2 today

related conditional

converse

$$q \rightarrow p$$

inverse

$$\sim p \rightarrow \sim q$$

contrapositive

$$\sim q \rightarrow \sim p$$

logically equivalent

cond + CP INV CONV

Sep 26-4:00 PM

**KeyConcept** Conditional Statement

Words	Symbols
An <b>if-then statement</b> is of the form <i>if p, then q</i> .	$p \rightarrow q$ read <i>if p then q</i> , or <i>p implies q</i>
The <b>hypothesis</b> of a conditional statement is the phrase immediately following the word <i>if</i> .	$p$
The <b>conclusion</b> of a conditional statement is the phrase immediately following the word <i>then</i> .	$q$

p107

Sep 26-4:09 PM

If you live in Sioux Falls, then you live in SD. **T**  
 $p$                        $q$

**2 Related Conditionals** There are other statements that are based on a given conditional statement. These are known as **related conditionals**.

KeyConcept Related Conditionals		
Words	Symbols	Examples
A conditional statement is a statement that can be written in the form <i>if p, then q</i> .	$p \rightarrow q$	If $m\angle A$ is 35, then $\angle A$ is an acute angle.
The <b>converse</b> is formed by exchanging the hypothesis and conclusion of the conditional.	$q \rightarrow p$	If $\angle A$ is an acute angle, then $m\angle A$ is 35.
The <b>inverse</b> is formed by negating both the hypothesis and conclusion of the conditional.	$\sim p \rightarrow \sim q$	If $m\angle A$ is <i>not</i> 35, then $\angle A$ is <i>not</i> an acute angle.
The <b>contrapositive</b> is formed by negating both the hypothesis and the conclusion of the converse of the conditional.	$\sim q \rightarrow \sim p$	If $\angle A$ is <i>not</i> an acute angle, then $m\angle A$ is <i>not</i> 35.

F }  
 F }  
**T**

If you live in Sioux Falls, then you live in SD.

If it is Christmas Day, then it is December.

Sep 26-4:14 PM

A conditional and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional are either both true or both false. Statements with the same truth values are said to be **logically equivalent**.

KeyConcept Logically Equivalent Statements
<ul style="list-style-type: none"> <li>A conditional and its contrapositive are logically equivalent.</li> <li>The converse and inverse of a conditional are logically equivalent.</li> </ul>

Sep 26-4:15 PM

**Example 2** Write each statement in if-then form

- 5. Sixteen-year-olds are eligible to drive.
- 6. Cheese contains calcium.
- 7. The measure of an acute angle is between 0 and 90.
- 8. Equilateral triangles are equiangular.

→ If 16 then eligible to drive  
 iff  
 S. T If <sup>4</sup> equil then also equiangular  
 T If angles then sides  
 T If not = sides then not  $\angle$   
 T If not =  $\angle$  then not = sides

Sep 27-12:26 PM

**Example 3** Determine the truth value of each conditional statement. If true, explain your reasoning. If false, give a counterexample.

- 10. If  $x^2 = 16$ , then  $x = 4$ . **F** What if  $x = -4$ ?
- 11. If you live in Charlotte, then you live in North Carolina. **T**
- 12. If tomorrow is Friday, then today is Thursday. **F** Mi.
- 13. If an animal is spotted, then it is a Dalmatian. **F** CE cheetah
- 14. If the measure of a right angle is 95, then bees are lizards. **T**
- 15. If pigs can fly, then  $2 + 5 = 7$ . **T**

Remember: benefit of the doubt...

Sep 27-12:27 PM

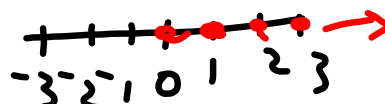
Whiteboards

**Example 4** **CSSE ARGUMENTS** Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

16. If a number is divisible by 2, then it is divisible by 4.

17. All whole numbers are integers

cond. if whole then integer T  
 CV if integer then whole  
 inv if not whole, not integer F CE = -3  
 CP if not int then not whole T F CE = -5  
 • Start by writing in if/then form.



Sep 27-12:27 PM

T If a triangle is equilateral, then each angle is 60 degrees.

T If a triangle has three 60 degree angles, then it is equilateral.

biconditional  $p \leftrightarrow q$   
 If and only if...iff all rel. conditionals are T

Sep 19-4:20 PM

**Key Concept** Biconditional Statement

**Words** A biconditional statement is the conjunction of a conditional and its converse.

**Symbols**  $(p \rightarrow q) \wedge (q \rightarrow p) \rightarrow (p \leftrightarrow q)$ , read *p if and only if q*

                                            
 T            T      *If and only if can be abbreviated iff.*

Sep 19-4:19 PM

**Examples**

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

a. An angle is a right angle if and only if its measure is 90. **T** if meas is 90 then rt  $\angle$   
 Conditional: If an angle measures 90, then the angle is right.  
 Converse: If an angle is right, then the angle measures 90. **T** if rt  $\angle$  then meas is 90  
 Both the conditional and the converse are true, so the biconditional is true.

b.  $x > -2$  iff  $x$  is positive.  
 Conditional: If  $x$  is positive, then  $x > -2$ .  
 Converse: If  $x > -2$ , then  $x$  is positive. **F**  
**T** Let  $x = -1$ . Then  $-1 > -2$ , but  $-1$  is not positive. So, the biconditional is false.

Write both statement & converse. Are they both true?

Sep 27-12:28 PM

## Exercises

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

1. Two angles are complements if and only if their measures have a sum of 90. T

2. There is no school if and only if it is Saturday.

1.

T If 2  $\angle$ s comp then sum is 90

T If 2  $\angle$ s sum 90 then comp

2. If Sat no school T } F  
If no school then Sat F }

Sep 27-12:29 PM

Sep 30-7:29 PM