Geometry 2.3

Quiz 2.1-2.2 is Wed.

Analyze statements in if-then form
Write the converse, inverse, and contrapositive of conditional statements

conditional statement — if $\boxed{\text{today is T}}^{p}$ then $\boxed{\text{tomm will be Wed.}}^{q}$

hypothesis

conclusion

related conditional — if p then q

converse

inverse — $p \rightarrow q$

contrapositive

logically equivalent — the same Truth val.

Activ: If you give a mouse a cookie (if time)

Sep 26-4:00 PM

It isn't about the order in the sentence...it's about what it *means*!

KeyConcept Conditional Statement

Words	Symbols
An **if-then statement** is of the form *if p, then q.*	$p \rightarrow q$ read *if p, then q,* or *p implies q*
The **hypothesis** of a conditional statement is the phrase immediately following the word *if.*	p
The **conclusion** of a conditional statement is the phrase immediately following the word *then.*	q

P: It is Xmas

Q: it is Dec.

$p \rightarrow q$

If it is Christmas, then it is December.

(It is December, if it's Christmas.)

Sep 26-4:09 PM

Example 1 Identify the Hypothesis and Conclusion

Identify the hypothesis and conclusion of each conditional statement.

a. If the forecast is rain, then I will take an umbrella.

H C

b. A number is divisible by 10 if its last digit is a 0.

C H

Where is the "if" (cause)?

Where is the "then" (effect)?

(note: might be at the beginning, middle, or end of the sentence)

Guided Practice

H C

1A. If a polygon has six sides, then it is a hexagon.

1B. Another performance will be scheduled if the first one is sold out.

C H

Sep 26-4:09 PM

effect cause

Points will be deducted from any paper turned in after Wednesday's deadline.

Conclusion Hypothesis

If a paper is turned in after Wednesday's deadline, then points will be deducted.

Remember, the conclusion depends upon the hypothesis.

Sep 26-4:10 PM

Example 2 Write a Conditional in If-Then Form

Identify the hypothesis and conclusion for each conditional statement. Then write the statement in if-then form.

a. A mammal is a warm-blooded animal.

conclusion (outcome)
depends on hypothesis (cause)

If mammal then warm-blooded.
H C

b. A prism with bases that are regular polygons is a regular prism.

Note: Try writing in if/then format
mathematical definitions will often work both ways "iff"

iff (if + only if)

Sep 26-4:10 PM

GuidedPractice

2A. Four quarters can be exchanged for a $1 bill.

2B. The sum of the measures of two supplementary angles is 180.

Sep 26-4:11 PM

Example 3 Truth Values of Conditionals

Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

a. If you divide an integer by another integer, the result is also an integer.

F CE: 5÷3 not an int.

b. If next month is August, then this month is July.

???!?!?!?

c. If a triangle has four sides, then it is concave. T

GuidedPractice

3A. If $\angle A$ is an acute angle, then $m\angle A$ is 35.

F CE: could be 85°

3B. If $\sqrt{x} = -1$, then $(-1)^2 = -1$. T

??!!?

Sep 26-4:13 PM

Notice that a conditional is false *only* when its hypothesis is true and its conclusion is false.

Conditional Statements		
p	q	$p \rightarrow q$
T	T	T→T T
T	F	T→F F
F	T	F→T T
F	F	F→F T

Notice too that when a hypothesis is false, the conditional will *always* be considered true, regardless of whether the conclusion is true or false.

To show that a conditional is true, you must show that for each case when the hypothesis is ... onal is also true. To show that a conditional is false, you only need to fi... ample.

WatchOut!

Analyzing Conditionals

When analyzing a conditional, do not try to determine whether the argument makes sense. Instead, analyze the form of the argument to determine whether the conclusion follows logically from the hypothesis.

??!!?

Sep 26-4:12 PM

The hypothesis and the conclusion of a conditional statement can have a truth value of true or false, as can the conditional statement itself. Consider the following conditional.

p q

If Tom finishes his homework, then he will clean his room.

Hypothesis	Conclusion	Conditional	
Tom finishes his homework.	Tom cleans his room.	If Tom finishes his homework, then he will clean his room.	
T	T	T	If Tom *does* finish his homework and he *does* clean his room, then the conditional is true.
T	F	F	If Tom does *not* clean his room after he *does* finish his homework, then he has not fulfilled his promise and the conditional is false.
F	T	T	The conditional only indicates what will happen if Tom *does* finish his homework. He could clean his room or not clean his room if he does *not* finish his homework.
F	F	? T	

"benefit of the doubt"

When the hypothesis of a conditional is not met, the truth of a conditional cannot be determined. When the truth of a conditional statement cannot be determined, it is considered true by default.

Sep 26-4:12 PM

2 Related Conditionals There are other statements that are based on a given conditional statement. These are known as **related conditionals**.

KeyConcept Related Conditionals

Words	Symbols	Examples
A conditional statement is a statement that can be written in the form *if p, then q*.	$p \to q$	If $m\angle A$ is 35, then $\angle A$ is an acute angle.
The **converse** is formed by exchanging the hypothesis and conclusion of the conditional.	$q \to p$	If $\angle A$ is an acute angle, then $m\angle A$ is 35.
The **inverse** is formed by negating both the hypothesis and conclusion of the conditional.	$\sim p \to \sim q$	If $m\angle A$ is *not* 35, then $\angle A$ is *not* an acute angle.
The **contrapositive** is formed by negating both the hypothesis and the conclusion of the converse of the conditional.	$\sim q \to \sim p$	If $\angle A$ is *not* an acute angle, then $m\angle A$ is *not* 35.

$P \to Q$ T

$Q \to P$ F

$\sim P \to \sim Q$ F

$\sim Q \to \sim P$ T

If it is Christmas, then it is December. T

p q

p.109

CV if it's Dec, then X mas. F

I if isn't XMas then isn't Dec. F

CP if isn't Dec then isn't Xmas T

Sep 26-4:14 PM

A conditional and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional are either both true or both false. Statements with the same truth values are said to be **logically equivalent**.

KeyConcept Logically Equivalent Statements

- A conditional and its contrapositve are logically equivalent.
- The converse and inverse of a conditional are logically equivalent.

Sep 26-4:15 PM

GuidedPractice

Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

4A. Two angles that have the same measure are congruent.

4B. A hamster is a rodent.

1. Write in if/then form

2. Write the requested related conditional

3. Answer the question

Cond. if 2 ∠'s have same meas. then they are ≅ T

h c

cv If 2 ∠'s are ≅ then have same meas. T

INV if 2 ∠s dont have same meas then they are not ≅ T

CP if 2 ∠'s not ≅ then dont have same meas. T

Sep 26-4:15 PM

Sep 26-4:10 PM