

## Geometry 12.3

Find lateral areas and surface areas of pyramids.

Find lateral areas and surface areas of cones.

pyramid

apex/vertex

regular pyramid

base  $\rightarrow$  polygon

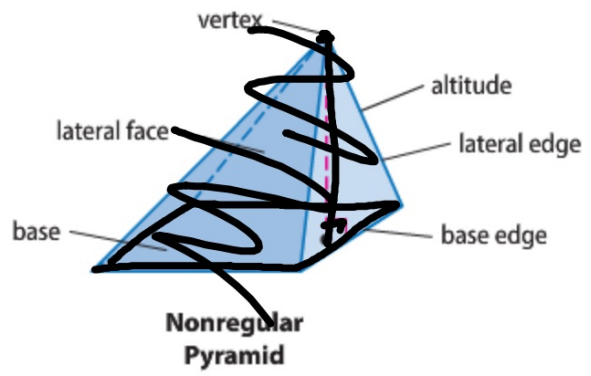
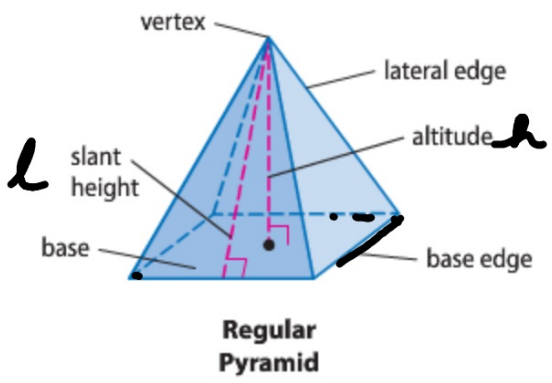
face  $\rightarrow$  triangles a face

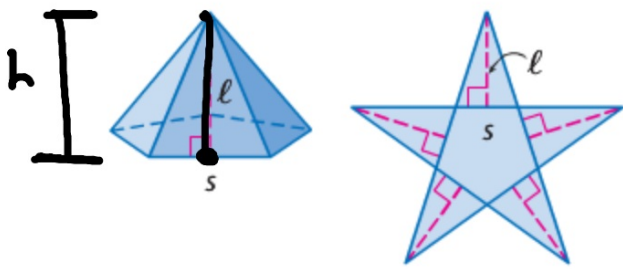
slant height base vertex  $\perp$

right cone

oblique cone







popcorn box

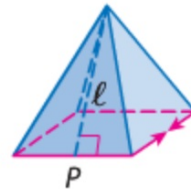
$$LA = p \cdot h \text{ rect. faces}$$



**KeyConcept** Lateral Area of a Regular Pyramid

**Words** The lateral area  $L$  of a regular pyramid is  $L = \frac{1}{2}P\ell$ , where  $\ell$  is the slant height and  $P$  is the perimeter of the base.

**Model**



**Symbols**  $L = \frac{1}{2}P\ell$

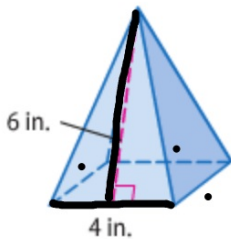
$$LA = \frac{1}{2} P \ell \quad \Delta \text{ faces}$$

Prisms  $LA = pl$ ...sides are rectangles

Pyramids  $LA = \frac{1}{2}pl$ ...sides are triangles

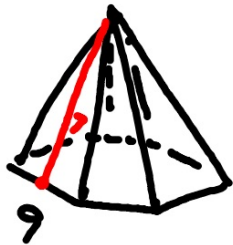
### Example 1 Lateral Area of a Regular Pyramid

Find the lateral area of the square pyramid.



$$\begin{aligned} LA &= \frac{1}{2} p \cdot l \\ &= \frac{1}{2} \cdot 16 \cdot 6 \\ &= 48 \text{ in}^2 \end{aligned}$$

1. Find the lateral area of a regular hexagonal pyramid with a base edge of 9 centimeters and a lateral height of 7 centimeters.




$$LA = \frac{1}{2} P \cdot \ell$$

$$= \frac{1}{2} \cdot 54 \cdot 7$$

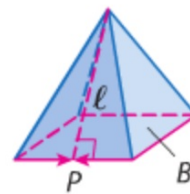
$$= 189 \text{ cm}^2$$

$$SA = LA + B$$

 **Key Concept** Surface Area of a Regular Pyramid

**Words** The surface area  $S$  of a regular pyramid is  $S = \frac{1}{2}P\ell + B$ , where  $P$  is the perimeter of the base,  $\ell$  is the slant height, and  $B$  is the area of the base.

**Model**

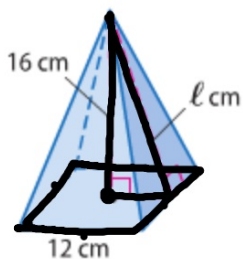


**Symbols**  $S = \frac{1}{2}P\ell + B$

$$= \frac{1}{2}P\ell + B$$

### Example 2 Surface Area of a Square Pyramid

Find the surface area of the square pyramid to the nearest tenth.



$$\begin{aligned}16^2 + 6^2 &= l^2 \\ 292 &= l^2 \\ l &= 17.089\end{aligned}$$



$$SA = LA + B$$

$$= \frac{1}{2} \cdot 48 \cdot 17.09 + 12 \cdot 12$$

$$= 410.16 + 144$$

$$= 554.2 \text{ cm}^2$$



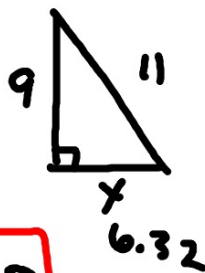
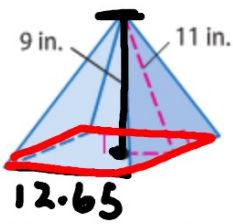
438.3 in<sup>2</sup>

Guided Practice

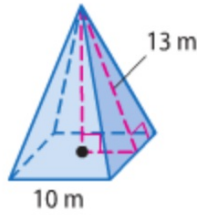
$$9^2 + x^2 = 11^2$$

$$x^2 = 40$$

2A.



2B.



$$SA = LA + B$$

$$= \frac{1}{2} \cdot 50.60 \cdot 11 +$$

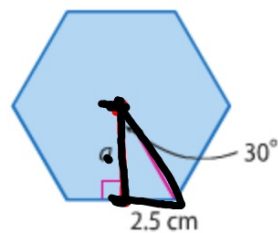
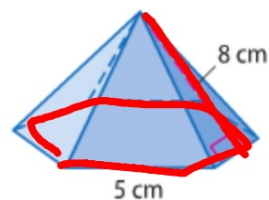
$$= 278.28 + 160.02$$

$$=$$

# SohcahToa

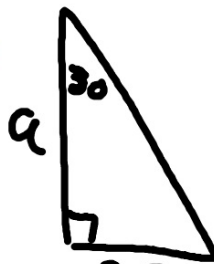
## Example 3 Surface Area of a Regular Pyramid

Find the surface area of the regular pyramid. Round to the nearest tenth.



$$A = \frac{1}{2} \text{apothem} \cdot p$$

$$= \frac{1}{2} \cdot 4.33 \cdot 30$$



$$\tan 30 = \frac{2.5}{a}$$

$$a(0.577) = 2.5$$

$$a = 4.33$$

$$SA = \frac{1}{2} p \cdot l + B$$

$$= \frac{1}{2} \cdot 30 \cdot 8 +$$

$$= 120 + 64.95$$

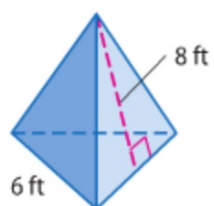
$$= 184.95$$

$$\text{cm}^2$$

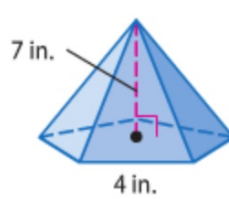
$$185.0$$

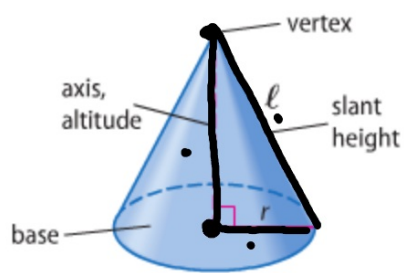
### Guided Practice

3A.

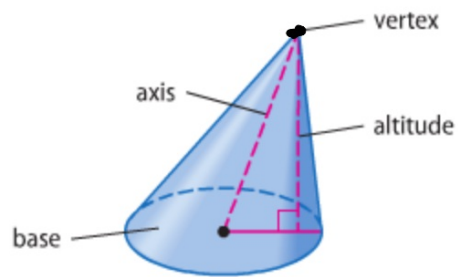


3B.





Right Cone



Oblique Cone

$$LA = \frac{1}{2} p l$$

$$SA = \frac{1}{2} p l + B$$

$$A = \pi r^2 \quad C = \pi d$$



**Key Concept** Lateral and Surface Area of a Cone

Words

The lateral area  $L$  of a right circular cone is

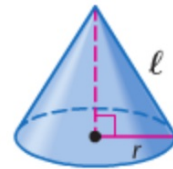
$$LA = \frac{1}{2} p l$$

The surface area  $S$  of a right circular cone is

$$SA = \frac{1}{2} p l + B$$

Symbols

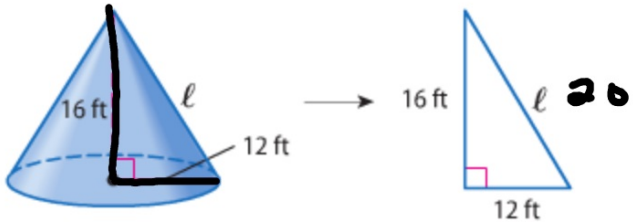
Model



**Real-World Example 4** Lateral Area of a Cone



**ARCHITECTURE** The ~~conical~~ slate roof at the right has a height of 16 feet and a radius of 12 feet. Find the lateral area.



$$LA = \frac{1}{2} p \cdot l$$
$$= \frac{1}{2} \pi \cdot 24 \cdot 20$$
$$12^2 + 16^2 = l^2$$
$$400 = l^2$$
$$20 = l$$

$$7539822$$
$$754.0 \text{ ft}^2$$

**Guided Practice**

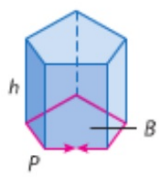
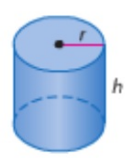
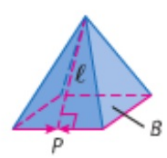
4. **ICE CREAM** A waffle cone is  $5\frac{1}{2}$  inches tall and the diameter of the base is  $2\frac{1}{2}$  inches. Find the lateral area of the cone. Round to the nearest tenth.

12.3 p. 854  
7-210  
55, 56, 57

**ConceptSummary** Lateral and Surface Areas of Solids

**WatchOut!**

**Bases** The bases of right prisms and right pyramids are not always regular polygons.

Solid	Model	Lateral Area	Surface Area
prism		$L = Ph$	$S = L + 2B$ or $S = Ph + 2B$
cylinder			
pyramid		$L = \frac{1}{2}P\ell$	$S = \frac{1}{2}P\ell + B$
cone	