Algebra 2 7.8

Use logarithms to solve problems involving exponential growth and decay

Use logarithms to solve problems of logistic growth

logistic

- half-life → time
- carbon dating

$e \to \ln$

$10 \to \log$

$(\ln e)^{2x}$

$1000 \to 500 \to 250 \to 125 \to$

C_{12}

C_{14}

The exponential growth equation $y = ae^{kt}$ is identical to the continuously compounded interest formula you learned in Lesson 7-7.

Continuous Compounding

$$A = Pe^{rt}$$

P **= initial amount**
A **= amount at time** ***t***
r **= interest rate**

Population Growth

$$y = ae^{kt}$$

a **= initial population**
y **= population at time** ***t***
k **= rate of continuous growth**

Real-World Example 3 Continuous Exponential Growth

POPULATION In 2007, the population of the state of Georgia was 9.36 million people. In 2000, it was 8.18 million.

a. Determine the value of k, Georgia's relative rate of growth.

(2007, 9.36)
(2000, 8.18)

$y = ae^{kt}$

$\frac{9.36}{8.18} = \frac{8.18e^{k \cdot 7}}{8.18}$

$\ln(1.1443) = \ln e^{7k}$

$\frac{0.1348}{7} = \frac{7k}{7}$

$K \approx 0.01925$

0.0193

$y = 8.18e^{0.0193t}$

2020

12.03

$\approx$ 12 million

b. When will Georgia's population reach ~~12~~ 15 million people?

$$\frac{15}{8.18} = \frac{8.18e^{0.0193t}}{8.18}$$

$$\ln 1.8337 = \ln e^{0.0193t}$$

$$0.6064 = 0.0193t$$

$$31.4 = t$$

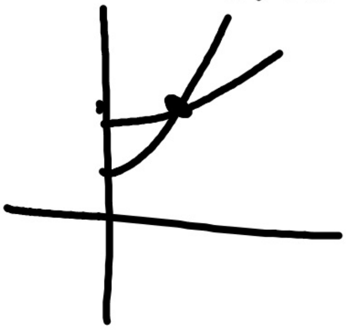
2031

?

c. **Michigan's population in 2000 was 9.9 million and can be modeled by $y = 9.9e^{0.0028t}$. Determine when Georgia's population will surpass Michigan's.**

GA

$$8.18e^{0.0193t} > 9.9e^{0.0028t}$$



$$0.0193t(2.1069) > 0.028t(2.2953)$$

$$0.04066t > 0.0643t$$

$$-0.02364t > 0$$

G>M

GuidedPractice

3. **BIOLOGY** A type of bacteria is growing exponentially according to the model $\boxed{y = 1000e^{kt},}$ where t is the time in minutes.

   A. If there are 1000 cells initially and 1650 cells after 40 minutes, find the value of k for the bacteria.

   B. Suppose a second type of bacteria is growing exponentially according to the model $y = 50e^{0.0432t}$. Determine how long it will be before the number of cells of this bacteria exceed the number of cells in the other bacteria.

$$\frac{1650}{1000} = \frac{1000e^{k \cdot 40}}{1000}$$

B>A

$$\ln \quad 1.65 = e^{40k}$$

$$0.5008 = 40k$$

$$0.0125 = k$$

$$y_A = 1000e^{0.0125t}$$

$$y_B = 50e^{0.0432t}$$

$$50e^{0.0432t} > 1000e^{0.0125t}$$

(increase/decrease)

KeyConcept Exponential Growth and Decay

Exponential Growth	Exponential Decay
Exponential growth can be modeled by the function $\boxed{f(x) = ae^{kt}}$, where a is the initial value, t is time in years, and k is a constant representing the rate of continuous growth.	Exponential decay can be modeled by the function $\boxed{f(x) = ae^{-kt}}$, where a is the initial value, t is time in years, and k is a constant representing the rate of continuous decay.

half-life = length of TIME

If you start with 100%... $\rightarrow$ 50%

Real-World Example 1 Exponential Decay

SCIENCE The half-life of a radioactive substance is the time it takes for half of the atoms of the substance to disintegrate. The half-life of Carbon-14 is 5730 years. Determine the value of *k* and the equation of decay for Carbon-14.

$$\frac{50}{100} = \frac{100}{100} e^{k \cdot 5730}$$

$$.5 = e^{5730k}$$

$$-0.6931 = 5730k$$

$$k = -0.000121$$

$$= ae^{-0.000121t}$$

GuidedPractice

1. The half-life of Plutonium-239 is 24,000 years. Determine the value of k.

$$50 = 100e^{k \cdot 24,000} \qquad ae^{-0.0000289t}$$

$$.5 = e^{24000k}$$

$$-0.6931 = 24000k$$

$$k = -0.0000289$$

So if it was 100% then...

Real-World Example 2 Carbon Dating

SCIENCE A paleontologist examining the bones of a prehistoric animal estimates that they contain 2% as much Carbon-14 as they would have contained when the animal was alive.

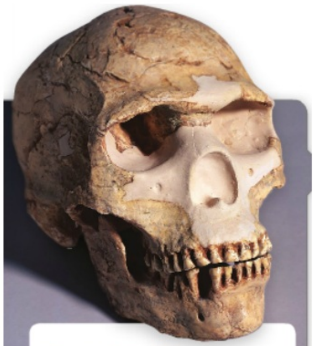
a. How long ago did the animal live? (Use k=-.00012 from ex. 1)

$$2 = 100e^{-0.00012t}$$

$$\ln 0.02 = e^{-0.00012t}$$

$$-3.912 = -0.00012t$$

$$t = 32{,}600$$



Real-WorldLink

The oldest modern human fossil, found in Ethiopia, is approximately 160,000 years old.

Source: National Public Radio

b. If prior research points to the animal being around 20,000 years old, how much Carbon-14 should be in the animal?

StudyTip

Carbon Dating When given a percent or fraction of decay, use an original amount of 1 for *a*. (100%)

$$x = 100\, e^{20{,}000(-0.00012)}$$

$$\approx 9\%$$

7. 8

21-550