VA HA

Algebra 2 8.4

Graph rational functions with vertical and horizontal asymptotes

x = y =

Graph rational functions with oblique asymptotes

Graph rational functions with point discontinuity

hole

$$y = \frac{1x+3}{x-5}$$

rational function

zero (of a function) x-int

vertical asymptote VA

Slant SA $y = mx + B$

horizontal asymptote HA

oblique (slant) asymptote SA

point discontinuity

1 Vertical and Horizontal Asymptotes

A **rational function** has an equation of the form $f(x) = \frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomial functions and $b(x) \neq 0$.

KeyConcept Vertical and Horizontal Asymptotes

Words If $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1, and $b(x) \neq 0$, then:

- $f(x)$ has a vertical asymptote whenever $b(x) = 0$. denom
- $f(x)$ has at most one horizontal asymptote.
  - If the degree of $a(x)$ is greater than the degree of $b(x)$, there is no horizontal asymptote.
  - If the degree of $a(x)$ is less than the degree of $b(x)$, the horizontal asymptote is the line $y = 0$.
  - If the degree of $a(x)$ equals the degree of $b(x)$, the horizontal asymptote is the line $y = \frac{\text{leading coefficient of } a(x)}{\text{leading coefficient of } b(x)}$. (ratio)

$\frac{x^2+3}{x-5}$

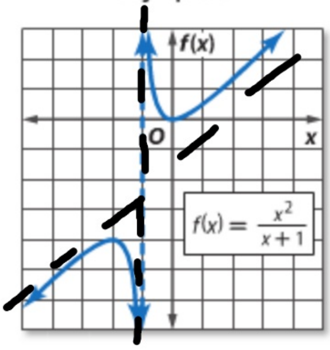
$\frac{x+4}{x^3}$

$\frac{3x^2}{5x^2}$

* complication point discontinuity

Examples

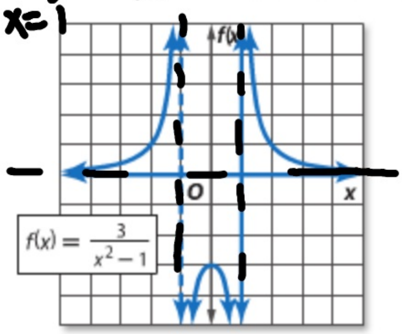
No horizontal asymptote



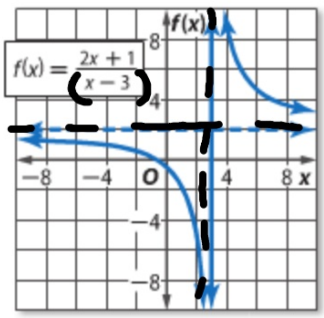
Vertical asymptote:
$x = -1$

$x^2 - 1$
$(x-1)(x+1)$
$x = -1$
$x = 1$

One horizontal asymptote



Vertical asymptotes:
$x = -1$, $x = 1$
Horizontal asymptote:
$f(x) = 0$



$\frac{2x}{1x}$

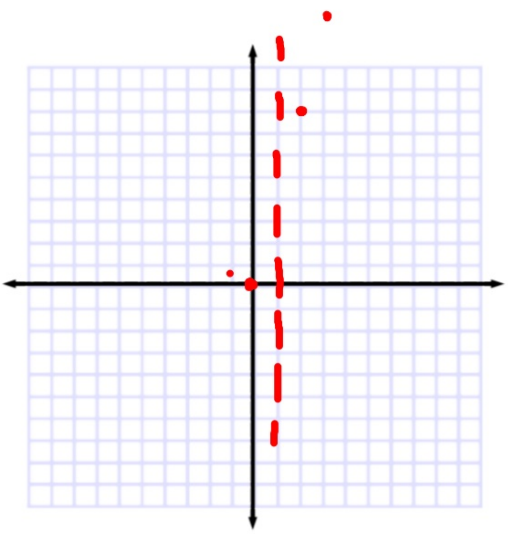
Vertical asymptote:
$x = 3$
Horizontal asymptote:
$f(x) = 2$

Example 1 Graph with no Horizontal Asymptote

Graph $f(x) = \frac{x^3}{(x-1)}$.   HA: no

$x - 1 = 0$
$x = 1$

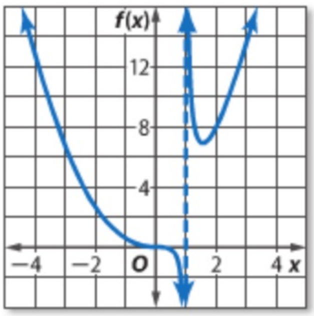
1. find the vertical asymptotes (denom=0)
2. find the horizontal asymptotes (if any)
   ...you can tell by degree of num/denom
3. Plot a few ordered pairs (each region)
   (find zeros (where numer=0))



	$\frac{x^3}{x-1}$	
-1	$\frac{-1}{-2}$	0.5
0	$\frac{0}{-1}$	0
2	$\frac{8}{1}$	8
3	$\frac{27}{2}$	13.5

next

x	$f(x)$
−3	6.75
−2	2.67
−1	0.5
0	0
0.5	−0.25
1.5	6.75
2	8
3	13.5



2 Oblique Asymptotes and Point Discontinuity

An **oblique asymptote**, sometimes called a *slant asymptote*, is an asymptote that is neither horizontal nor vertical.

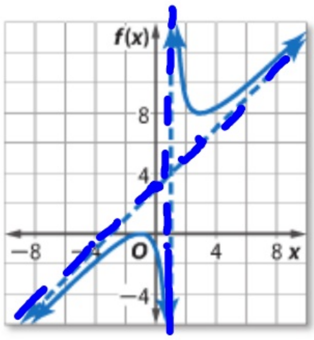
KeyConcept Oblique Asymptotes

Words If $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1 and $b(x) \neq 0$, then $f(x)$ has an oblique asymptote if the degree of $a(x)$ minus the degree of $b(x)$ equals 1. The equation of the asymptote is $f(x) = \frac{a(x)}{b(x)}$ with no remainder.

Example $f(x) = \frac{x^4 + 3x^3}{x^3 - 1}$ degree + 1

Vertical asymptote: $x = 1$

Oblique asymptote: $f(x) = x + 3$



Horizontal or slant (not both)

Long division ans

$$x + 3 + \cancel{\frac{x+3}{x^3-1}}$$

$$x^3 - 1 \overline{\big) x^4 + 3x^3}$$

$$-x^4 + x$$

$$3x^3 + x$$

$$-3x^3 + 3$$

$$x + 3$$

$y = x + 3$

GuidedPractice

Graph each function.

1A. $f(x) = \dfrac{x^2 - x - 6}{x + 1}$ *

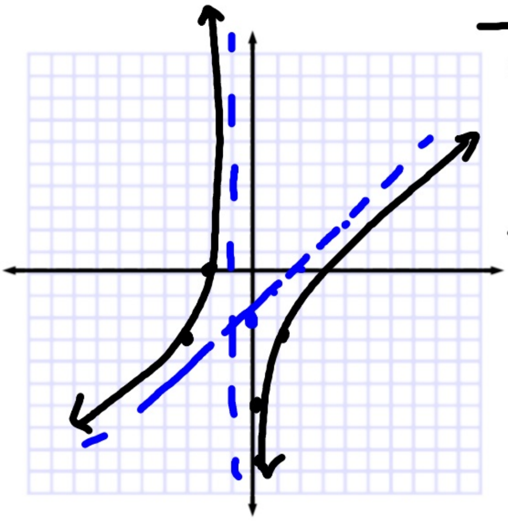
SA $y = x - 2$

$$\begin{array}{r|rrr} -1 & 1 & -1 & -6 \\ & \downarrow & -1 & 2 \\ \hline & 1 & -2 & -4 \end{array}$$

$$x - 2 \quad \frac{-4}{x+1}$$

$$x+1 \overline{\big)\, x^2 - x - 6}$$

$$\begin{array}{r} -x^2 - x \\ \hline -2x - 6 \\ +2x + 2 \\ \hline -4 \end{array}$$



0	$\frac{0-0-6}{1}$
1	$\frac{1-1-6}{2}$
-2	$\frac{4+2-6}{-1}$
-3	$\frac{9+3-6}{-2}$

1. find the vertical asymptotes
2. find the horizontal asymptotes (if any)
3. find the slant asymptotes (if any)
4. Consider zeros: Plot a few ordered pairs (each region)

Example 3 Determine Oblique Asymptotes

Graph $f(x) = \dfrac{x^2 + 4x + 4}{2x - 1}$

$\frac{1}{2}x + 2.25$

$2x-1 \,\overline{\big)\, x^2 + 4x + 4}$

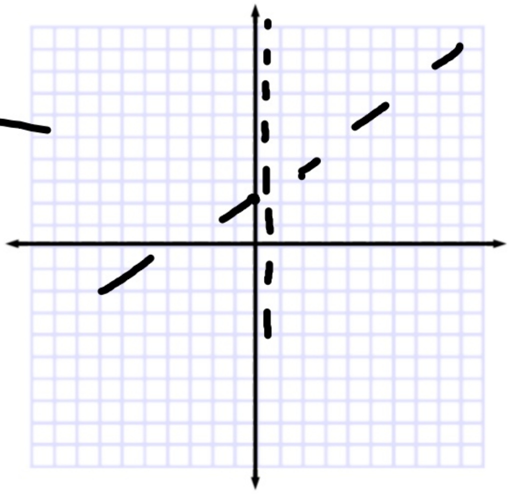
$-x^2 + \frac{1}{2}x$

$4.5x + 4$

$-4.5 + 2.25$

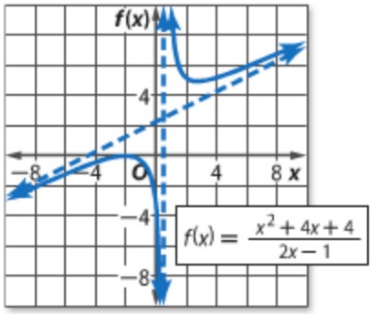
0

$2(?) = 4.5$



1. find the vertical asymptotes
2. find the horizontal asymptotes (if any)
3. find the slant asymptotes (if any)
4. Consider zeros: Plot a few ordered pairs (each region)

Draw the asymptotes, and then use a table of values to graph the function.



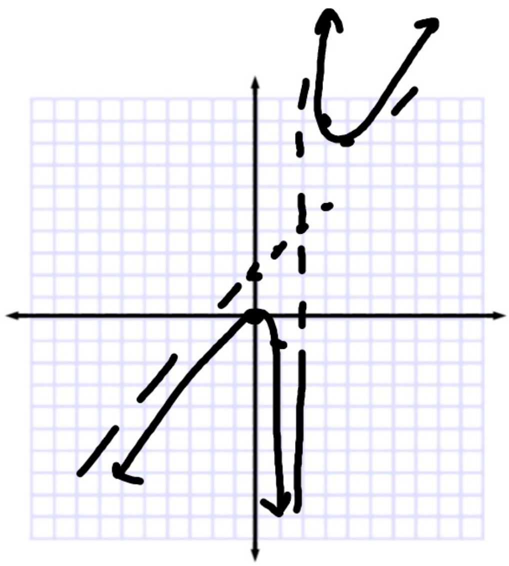
GuidedPractice

Graph each function.

3A. $f(x) = \frac{x^2}{x-2}$

$$
\begin{array}{r|l}
 & x + 2 \\
x - 2 & x^2 \\
 & -x^2 + 2x \\
\hline
 & 2x
\end{array}
$$

0	$\frac{0}{-2}$	
1	$\frac{1}{-1}$	-1
3	$\frac{9}{1}$	
4	$\frac{16}{2}$	8



1. find the vertical asymptotes
2. find the horizontal asymptotes (if any)
3. find the slant asymptotes (if any)
4. Consider zeros: Plot a few ordered pairs (each region)

$y = x + 2$

8
4 2
6

$$y = \frac{x^2 + 6x + 8}{x+4} = \frac{\cancel{(x+4)}(x+2)}{\cancel{(x+4)}}$$

point discontinuity

$y = x + 2$